ENTRETIEN DES CONNAISSANCES

1. Suppressions de parenthèses

* $(a+5)-(a+3) = a+5-a-3 = 2$ ✓
* $-(x+y)-(-x-y) = x-y+x+y = 0$ ✓
* $(2-b)-(a-1)-(-b+4) = 2-b-a+1+b-4 = -a$ ✓
* $-[a-(2+b)] = -[a+2-b] = -a+2+b$ ✓
* $a-[2b-(1-c)-3] = a-[2b-1+c-3] = a-2b+1-c+3 = a-2b-c+4$ ✓

  $=$ .....
* $-[a-(b-1)]+[1-(c+2)+4] = -[a-b+1]+[1-c-2+4]$

  $= -a+b-1+1-c-2+4 = -a+b-c+2$
* $\frac{a-3}{4} - \frac{2a-5}{3} = \frac{3a-9}{12} - \frac{8a-20}{12} = \frac{(3a-9)-(8a-20)}{12} = \frac{3a-9-8a+20}{12} = \frac{-5a+11}{12}$ ✓
* $-\frac{2+x}{3} - \frac{x}{4} + \frac{1}{5} = -\frac{40+20x}{60} - \frac{15x}{60} + \frac{12}{60} = \frac{-28+5x}{60}$

2. Distributivité

a) simple distributivité :

* $a(b+c-d) = ab+ac-ad$ ✓
* $6a(2b-3a) = 12ab-18a^2$ ✓
* $(ab+bc)2a^2 = 2a^3b+2a^2bc$ ✓
* $-x^2(2y^2-x) = -2x^2y^2+x^3 = -2x^2y^2+x^3$
* $(2a+3b)3a-(a^2+5ab) = 6a^2+9ab-a^2-5ab = 5a^2+4ab$

b) double distributivité :

* $(-x+a)(x+b) = -x^2-bx+ax+ab$ ✓
* $(2a+4b)(5a-2b) = 10a^2-4ab+20ab-8b^2 = 10a^2-8b^2+16ab$ ✓
* $(3ab-2)(4a-6) = 12a^2b-18ab-8a+12$ ✓
* $(x^2-3x-2)(x+1) = x^3+x^2-3x^2-3x-2x-2 = x^3-2x^2-5x-2$ ✓
* $(a-b)(2c+3d)-(2a+4)(c-3d) = 2ac+3ad-2bc-3bd-2ac+6ad-4c+12d$

  $= 9ad-2bc-3bd-4c+12d$ ✓
* $(2x-y)(a+2)-x+3y(2a-1) = 2ax+4x-ay-2y-x+6ay-3y$

  $= 3x+2ax-5y+5ay$ ✓

Corrigé !

c) produits remarquables :

* Applique la formule du carré d'une somme ou d'une différence :

$(x+y)^2 = x^2 + 2xy + y^2$ ✓

$(b+4)^2 = b^2 + 2b \cdot 4 + 4^2 = b^2 + 8b + 16$ ✓

$(2a+3c)^2 = (2a)^2 + 2 \cdot 2a \cdot 3c + (3c)^2 = 4a^2 + 12ac + 9c^2$ ✓

$(b-4)^2 = b^2 - 2b4 + 4^2 = b^2 - 8b + 16$ ✓

$(5c-4)^2 = (5c)^2 - 2 \cdot 5c \cdot 4 + 4^2 = 25c^2 - 40c + 16$ ✓

$(x-7)^2 = x^2 - 2x7 + 7^2 = x^2 - 14x + 49$ ✓

$(y+3z)^2 = y^2 + 2y \cdot 3z + 3z^2 = y^2 + 6yz + z^2$ ✓

$(3-c)^2 = 3^2 - 2 \cdot 3 \cdot c + c^2 = 9 - 6c + c^2$ ✓

$(2b+5c)^2 = (2b)^2 + 2 \cdot 2b \cdot 5c + (5c)^2 = 4b^2 + 20bc + 25c^2$ ✓

$(3x+6)^2 = (3x)^2 + 2 \cdot 3x \cdot 6 + 6^2 = 9x^2 + 36x + 36$ ✓

$(x^3-3)^2 = (x^3)^2 - 2 \cdot 3 \cdot x^3 + 3^2 = x^6 - 6x^3 + 9$ ✓

$(a^2+2c)^2 = (a^2)^2 + 2 \cdot a^2 \cdot 2c + (2c)^2 = a^4 + 4a^2c + 4c^2$ ✓

$(3b^2-5c)^2 = (3b^2)^2 - 2 \cdot 3b^2 \cdot 5c + (5c)^2 = 3b^4 - 30b^2c + 25c^2$ ✓

$(2ab+2b^2)^2 = (2ab)^2 + 2 \cdot 2b^2 \cdot 2ab + (2b^2)^2 = 4a^2b^2 + 8ab^3 + 4b$

$(4a^3-5c^2)^2 = (4a^3)^2 - 2 \cdot 4a^3 \cdot 5c^2 + (5c^2)^2 = 16a^6 - 40a^3c^2 + 25c^4$

$(b^2+4b^3)^2 = (b^2)^2 + 2 \cdot 4b^3 \cdot b^2 + (4b^3)^2 = b^4 + 8b^5 + 16b^6$ ✓

$(5y^2-3x^3)^2 = (5y^2)^2 - 2 \cdot 5y^2 \cdot 3x^3 + 3x^6 = 25y^4 - 30y^2x^3 + 9x^6$ ✓

$(7a^2+6c)^2 = (7a^2)^2 + 2 \cdot 7a^2 \cdot 6c + (6c)^2 = 49a^4 + 84a^2c + 36c^2$ ✓

$(3x^2y-2xy^2)^2 = (3x^2y)^2 - 2 \cdot 2xy^2 \cdot 3x^2y + (2xy^2)^2 = 9x^4y^2 - 12x^3y^3 + 4x^2y$

$(5a^3c+b^2)^2 = (5a^3c)^2 + 2 \cdot b^2 \cdot 5a^3c + (b^2)^2 = 25a^6c^2 + 10b^2a^3c + b^4$ ✓

*Applique la formule du produit de 2 binômes conjugués :

$(b-4)(b+4) = b^2 - 4^2 = b^2 - 16$ ✓

$(a-5)(a+5) = a^2 - 5^2 = a^2 - 25$ ✓

$(5b-2)(5b+2) = (5b)^2 - 2^2 = 25b^2 - 4$ ✓

$(a^2-3)(a^2+3) = a^4 - 3^2 = a^4 - 9$ ✓

$(3a-2)(3a+2) = 9a^2 - 4$ ✓

$(2a-c)(c+2a) = (2a)^2 - c^2 = 4a^2 - c^2$ ✓

corrigé!

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$(-b-4)(-b+5) = b^2 - 5b + 4b - 20 = b^2 - b - 20$ ✓

$(-4a+3)(4a+3) = \cancel{(16a^2 + 12a + 12a + 6) = 16a^2 + 24a + 6}$ $= (3-4a)\cdot(3+4a) = 9 - 16a^2$.

$(b^3+1)(b^3-1) = (b^3)^2 - 1 = b^6 - 1$

$(x^5+5)(x^5-5) = (x^5)^2 - 5^2 = x^{10} - 25$

*Identifie l'exercice en précisant s'il s'agit d'une double distributivité (DD), d'une simple distributivité (SD), d'une somme au carré (SC), d'une différence au carré (DC) ou d'un produit de deux binômes conjugués (BC). Ensuite, effectue-le.

CD ✓	$(3a-b)^2 =$	$(3a)^2 - 2\cdot 3a\cdot b + b^2$ ✓
DD ✓	$(x-2y)(x+y) =$	$x^2 + xy - 2xy - 2y^2 = x^2 - xy - 2y^2$ ✓
SD ✓	$2x(3xy-5) =$	$6x^2y - 10x$ ✓
BC ✓	$(2b-4)(4+2b) =$	$(2b)^2 - 4^2 = 4b^2 - 16$ ✓
SC ~~DD~~ ✓	$(-a-b)(a+b) =$	$-a^2 - ab - ab - b^2 = -a^2 - 2ab - b^2$ ✓
SC ✓	$(b+3)(b+3) =$	$b^2 + 2\cdot 3\cdot b + 3^2 = b^2 + 6 + 9 + 2b.$
BC ✓	$(3x+1)(-1+3x) =$	$(3x)^2 - 1^2 = 9x^2 - 1$ ✓
SD ✓	$-7(a-3b) =$	$-7a + 21b$ ✓
BC ✓	$(-3x-1)(3x-1) =$	$-(3x)^2 - 1^2 = -9x^2 + 1^2$ ✓
BC ✓	$(b-4)(-b-4) =$	$-b^2 + 4^2 = -b^2 + 16$ ✓
DC ✓	$(a^2-3c^3)^2 =$	$(a^2)^2 - 2\cdot a^2\cdot(3c^3)^2 = a^4 - 6a^2 - c^2 + 9^6$
BC ✓	$(4a+5b)(4a-5b) =$	$(4a)^2 - (5b)^2 = 16a^2 - 25b^2$ ✓
SC ✓	$(2a^2+3b^3)^2 =$	$4a^4 + 2\cdot 3b^3\cdot 2a^2 + (3b^3)^2 = 4a^2 + 12a^2 + 9b^6$
BC ✓	$(a^2+4b^3)(4b^3-a^2) =$	$16b^6 - a^4$ ✓

3. Calcul du P.G.C.D et du P.P.C.M.

a) calcule le P.G.C.D. et le P.P.C.M. de 160 et de 96

$160 \wedge 96 = 2^5 = 32$

$160 \vee 96 = 2^5 \cdot 3 \cdot 5 = 480$

160	2	96	2
80	2	48	2
40	2	24	2
20	2	12	2
10	2	6	2
5	5	3	3
1		1	

$160 = 2^5 \cdot 5$ $96 = 2^5 \cdot 3$

✓

b) calcule le P.G.C.D. et le P.P.C.M. de 990 et de 280

$990 \wedge 280 = 2\cdot 5 = 10$

$990 \vee 280 = 2 \cdot 3^2 \cdot 5 \cdot 7 \cdot 11$

$= 27 \cdot 720$

990	2	280	2
495	3	140	2
165	3	70	2
55	5	35	5
11	11	7	7
1		1	

$990 = 2\cdot 3^2 \cdot 5 \cdot 11$ $280 = 2^3 \cdot 5 \cdot 7$

✓

Corrigé!

4. Les fractions

a) Rends les fractions irréductibles. Veille à rendre positif le dénominateur de chaque fraction.

$\frac{12}{9} = \frac{4}{3}$ $\quad \frac{81}{90} = \frac{9}{10}$ $\quad \frac{-350}{650} = \frac{-7}{13}$ $\quad \frac{36}{252} = \frac{1}{7}$

$\frac{-9}{-6} = \frac{-3}{-2}$ $\quad \frac{-17}{-85} = \frac{1}{5}$ $\quad \frac{150}{-420} = \frac{-5}{14}$ $\quad \frac{112}{-26} = \frac{-48}{13}$

$\frac{8}{-6} = \frac{4}{-3}$ $\quad \frac{-45}{60} = \frac{-3}{4}$ $\quad \frac{-126}{-81} = \frac{-14}{9}$ $\quad \frac{10}{-30} = \frac{1}{-3}$

$\frac{-15}{3} = -5$ $\quad \frac{30}{-45} = \frac{-2}{3}$ $\quad \frac{121}{-55} = \frac{-11}{5}$ $\quad \frac{144}{-28} = \frac{-34}{7}$

Simplifie quand c'est possible les fractions suivantes

$\frac{6+4}{66+4} = \frac{1}{7}$ (impossible S.) $\quad \frac{3.3.3.3.5}{3.3.5} = 9$ (possible)

$\frac{121+11}{143-11} = \frac{132}{132} = 1$ (poss.) $\quad \frac{3^3}{3^5} = \frac{1}{9} = \frac{1}{3^2}$ (possible)

$\frac{6.7}{7.13} = \frac{6}{13}$ (possible) $\quad \frac{2.3^2}{2^3.3} = \frac{3}{2^2}$ (possible)

$\frac{4+3}{9+6} = \frac{7}{15}$ (impossible) $\quad \frac{11^2.6}{5.11} = \frac{11.6}{5} = \frac{66}{5}$ (possible)

$\frac{4.3}{9.6} = \frac{2}{9}$ (possible) $\quad \frac{5^3.2.7}{5^5.3} = \frac{2.7}{5^2.3} = \frac{14}{75}$ (possible)

$\frac{15.3}{15-3} = \frac{45}{12} = \frac{15}{4}$ (impossible) $\quad \frac{3^2.2}{1^4+2^3+3^2} = \frac{18}{18} = 1$ (impossible)

Rends les fractions suivantes irréductibles

$\frac{-9x^2}{12x^3} = \frac{-3}{4x^3}$ $\quad \frac{-12b}{-16b^3} = \frac{-3}{-4b^2}$ $\quad \frac{18ab^3}{21a^3b^4} = \frac{6}{7a^2b}$

Corrigé !

b) Opérations sur les fractions

Calcule (le résultat doit être une fraction irréductible).

$\frac{5}{7} + \frac{-4}{7} = \frac{1}{7}$

$\frac{-3}{4} + \frac{5}{6} = \frac{-9+10}{12} = \frac{1}{12}$

$\frac{3}{4} + \frac{-5}{+4} = \frac{3+-5}{4} = \frac{-2}{4} = \frac{-1}{2}$

$\frac{-1}{5} - \frac{-3}{4} = \frac{-4--15}{20} = \frac{11}{20}$

$\frac{+2}{+3} - \frac{1}{3} = \frac{2-1}{3} = \frac{1}{3}$

$\frac{4}{9} - \frac{5}{6} = \frac{8-15}{18} = \frac{-7}{18}$

$\frac{-11}{2} + \frac{-3}{+2} = \frac{-11+-3}{2} = \frac{-14}{2} = -7$

$-\frac{7}{10} + \frac{-4}{15} = -\frac{29}{30}$

$\frac{-3}{1} + \frac{-1}{5} = \frac{-15+-1}{5} = \frac{-16}{5}$

$\frac{-4}{1} - \frac{4}{3} = \frac{-12-4}{3} = \frac{-16}{3}$

$\frac{-3}{+4} + \frac{2}{1} = \frac{-3+8}{4} = \frac{5}{4}$

$\frac{2}{7} - \frac{7}{2} = \frac{4-49}{14} = \frac{45}{14}$

$\frac{-5}{2} + \frac{3}{4} = \frac{-10+3}{4} = \frac{-7}{4}$

$-\frac{5}{2} - \frac{5}{6} = -\frac{15-5}{6} = -\frac{10}{6} = -\frac{5}{3}$

Calcule le plus simplement possible.

$0{,}8 - \frac{1}{2} = \frac{8}{10} - \frac{5}{10} = \frac{3}{10}$

$\frac{1}{2} - 0{,}4 = \frac{5}{10} - \frac{4}{10} = \frac{1}{10}$

$-0{,}5 + \frac{1}{4} = -\frac{2}{4} + \frac{1}{4} = \frac{-1}{4}$

$0{,}25 - \frac{1}{2} = \frac{1}{4} - \frac{2}{4} = -\frac{1}{4}$

$\frac{-2}{7} + 0{,}75 = \frac{-2}{7} + \frac{3}{4} = \frac{13}{28}$

$\frac{1}{4} - 2{,}5 = 0{,}25 - 2{,}5 = -2{,}25 = \frac{-9}{4}$

$\frac{1}{3} + \frac{1}{6} = \frac{2+1}{6} = \frac{3}{6} = \frac{1}{2}$

$-\frac{2}{3} + 0{,}6 = -\frac{10}{15} + \frac{9}{15} = \frac{-1}{15}$

corrigé!

Calcule mentalement

$\frac{2}{3} + \frac{1}{3} = \frac{3}{3} = 1$ $\quad \frac{2}{3} \cdot \frac{1}{3} = \frac{2}{9}$ $\quad \frac{2+1}{3+3} = \frac{3}{6} = \frac{1}{2}$

$\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$ $\quad \frac{5}{2} : 2 = \frac{5}{2} \cdot \frac{1}{2} = \frac{5}{4}$ $\quad \frac{5}{2} + \frac{1}{2} = \frac{6}{2} = \frac{3}{1} = 3$

Calcule

$\frac{\frac{8}{35}}{\frac{2}{5}} = \frac{8}{35} : \frac{2}{5} = \frac{\cancel{8}^{4}}{\cancel{35}_{7}} : \frac{\cancel{5}^{1}}{\cancel{2}_{1}} = \frac{4}{7}$

$\frac{\frac{-5}{9}}{\frac{-7}{12}} = \frac{-5}{\cancel{9}_{3}} \cdot \frac{\cancel{12}^{4}}{-7} = \frac{-20}{-21}$

$\frac{\frac{3}{2}}{5} = \frac{3}{2} \cdot \frac{1}{5} = \frac{3}{10}$

$\frac{\frac{-12}{35}}{12} = \frac{-\cancel{12}}{35} \cdot \frac{1}{\cancel{12}} = \frac{-1}{35}$

$\frac{5}{\frac{-5}{3}} = 5 \cdot \frac{3}{-5} = -3$ A

$\frac{\frac{-3}{7}}{-7} = \frac{-3}{7} \cdot \frac{1}{-7} = \frac{-3}{-49}$

Calcule en respectant les règles de priorité des opérations.

Corrigé.

$\frac{3}{2} \cdot \frac{1}{5} + \frac{1}{2} = \frac{3}{10} + \frac{1}{2} = \frac{3}{10} + \frac{5}{10} = \frac{8}{10}$

$\frac{5}{4} - \frac{1}{3} \cdot \frac{1}{2} = \frac{5}{4} - \frac{1}{6} = \frac{30}{24} - \frac{4}{24} = \frac{26}{24} = \frac{13}{12}$

5. $(\frac{-1}{2})^3 + \frac{1}{4} = -\frac{5}{8} + \frac{1}{4} = \frac{-5}{8} + \frac{2}{8} = \frac{-3}{8}$

$3 - 2.(\frac{-1}{5})^2 =$ $3-2\cdot\left(-\frac{1}{5}\right)^2 = 3-2\cdot\frac{1}{25} = 3-\frac{2}{25} = \frac{75-2}{25} = \frac{73}{25}$

Réduis les expressions suivantes.

$\frac{a+2}{3} + \frac{a-5}{2} =$ $\frac{2a+4}{6} + \frac{3a-15}{6} = \frac{2a+4+3a-15}{6} = \frac{5a-11}{6}$

$\frac{2a-5}{4} - \frac{5-3a}{6} =$ $\frac{6a-15}{12} - \frac{10-6a}{12} = \frac{6a-15-10+6a}{12} = \frac{12a-25}{12} =$

$\frac{1+a}{9} - \frac{2a+5}{6} =$ $\frac{2\cdot(1+a)}{18} - \frac{3\cdot(2a+5)}{18} = \frac{2+2a}{18} - \frac{6a+15}{18} = \frac{2+2a-6a-15}{18} = \frac{-13-4a}{18}$

$2a + \frac{a-2}{3} =$ $\frac{6a}{3} + \frac{a-2}{3} = \frac{6a+a-2}{3} = \frac{7a-2}{3}$

$\frac{2a+2}{5} - a =$ $\frac{2a+2}{5} - \frac{5a}{5} = \frac{2a+2-5a}{5} = \frac{-3a+2}{5}$

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5. Equations

a) Résous les équations suivantes.

$y + 5 = -8$ $y = -13$	$5x + 2 = 3x + 8$ 3	$3.(x-2) = 2.(x+3)$ 12
$2x = -12$ $x = -6$	$z + 9 = 5z - 3$ 3	$-2(x+5) = 3.(x-2)$ $-\frac{4}{5}$
$-3t = 15$ -5	$3y - 5 = 4y - 7$ 2	$5 + (x-2) = 2 - (x+4)$ $\frac{-5}{2}$
$2v - 2 = 10$ 6	$v - 4 = -3v + 20$ 6	$-4.(2-5x) = 5 - (3-5x)$ $\frac{2}{3}$
$-5x + 1 = 11$ -2	$5 - 2z = -3z + 7$ 2	$2 - (x-3) - 2.(x+3) = 0$ $\frac{-1}{3}$

Résous les équations suivantes (x est différent de 0 quand il est diviseur)

$\frac{6}{10} = \frac{x}{20}$ 12	$\frac{-x}{21} = \frac{2}{-7}$ 6	$\frac{-x}{7} = \frac{-12}{21}$ 4
$\frac{25}{-30} = \frac{-50}{x}$ 60	$\frac{x}{-16} = \frac{-5}{16}$ 5	$\frac{-56}{48} = \frac{21}{-x}$ 18

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Résous les équations :

$(x+5)^2 = x.(x+5)$

$x^2+2x5+5^2 = x^2+5x$

$x^2+10x+25 = x^2+5x$

$x^2+10x-x^2-5x = -25$

$5x = -25$

$x = -5$

$(x-3).(x+3)=(x-3)^2$

$x^2-3^2 = x^2-2\cdot3\cdot x+3^2$

$x^2-9 = x^2-6x+9$

$x^2-x^2+6x = +9+9$

$6x = 18$

$x = +3.$

$(2x+3)^2=(2x-3)^2$

$4x^2+2\cdot2x\cdot3+3^2 = 4x^2-2\cdot3\cdot2x+3^2$

$4x^2+12x+9 = 4x^2-12x+9$

$+12x+12x = 0$

$24x = 0$

$3x.(5x-2)=(4x-1)^2-(x-2)^2$

$15x^2-6x = 16x^2-2\cdot4x+1^2-x^2-2\cdot2x+2^2$

$15x^2-6x = 16x^2-8x+1-x^2+4x-4$

$15x^2-6x = 15x^2+4x+5$

$-6x-4x = 5$

$+10x = -5$

$x = \frac{-1}{2}$

$(3x+4)^2=3x.(3x-5)$

$3x^2+2\cdot3x\cdot4+4^2 = 9x^2-15x$

$9x^2+24x+16 = 9x^2-15x$

$9x^2-9x+24x+15x = -16$

$39x = -16$

$x = \frac{-16}{39}$

$2x.(x-2)-(x+3).(2x+1)=0$

$(2x^2-4x)+2x^2+x+6x+3 = 0$

$2x^2-4x-2x^2-x-6x-3 = 0$

$-11x-3 = 0$

$-11x = +3$

$+x = -\frac{3}{11}$

Corrigé !

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